

A Hierarchical Approach for the Association Problem with Misleading Partial Channel State Information

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Abstract

In this paper, we develop a hierarchical Bayesian game framework where users compete to maximize their throughput by picking the best locally serving radio access network (RAN) with respect to their own measurement, their demand and a partial statistical channel state information (CSI) of other users. In particular, we investigate the properties of a Stackelberg game, in which the base station is a player on its own. We derive analytically the utilities related to the channel quality perceived by users to obtain the equilibria. We show by means of a Stackelberg formulation, how the operator, by sending appropriate information about the state of the channel, can optimize its global utility while users maximize their individual utilities. The proposed hierarchical decision approach for wireless networks can reach a good trade-off between the global network performance at the equilibrium and the requested amount of signaling. Typically, it is shown that when the network goal is orthogonal to user's goal, this can lead the users to a misleading association problem.

Index Terms

WLAN, 3G LTE, association problem, misleading information, channel state information, game theory, Bayes-Nash equilibrium, Bayes-Stackelberg equilibrium.

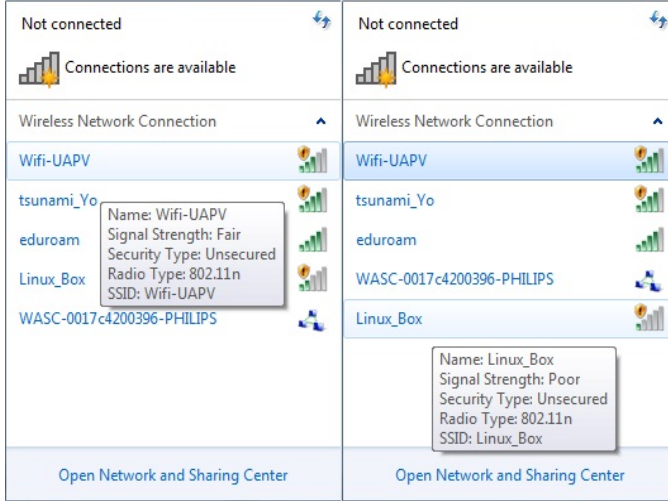


Fig. 1. The association problem of choice of the access point: Information available when taking a decision.

I. INTRODUCTION

Efficient design of wireless networks calls for end users implementing radio resource management (RRM), which requires knowledge of the mutual channel state information in order to limit the influence of interference impairments on the decision making. However, full CSI assumption is not always practical because communicating channel gains between different users in a time varying channel within the channel coherence time may lead to large overhead. In this case, it is more appropriate to consider each channel coherence time as a one-stage game where players are only aware of their own channel gains and their opponent's channel statistics (which vary slowly compared to the channel gains and, therefore, can be communicated [1]). The interaction between the players may be repeated but with a different and independent channel realization each time and therefore is not a repeated game. This motivates the use of games with incomplete information, also known as Bayesian games [2], [3] which have been incorporated into wireless communications for problems such as power control [4] and spectrum management in the interference channel [5]. In [4], a distributed uplink power control in a multiple access (MAC) fading channel was studied and shown to have a unique Nash equilibrium (NE) point. With the same incomplete information, it was shown [5] that in a symmetric interference channel with a one-time interaction, there exists a unique symmetric strategy profile which is a NE point. This result however is limited

to scenarios where all users statistically experience identical channel conditions (due to the symmetry assumption) and does not apply to interactions between weak and strong users.

In this paper, we generalize the analysis of [6] to the multi-user case and present an alternative approach for improving the network efficiency by introducing a certain degree of hierarchy between the users and the base station. More specifically, we propose a Stackelberg formulation of the association problem when a partial channel state information is assumed at the transmitter. By Stackelberg we mean distributed decision making assisted by the network, where the wireless users aim at maximizing their own utility, guided by aggregated information broadcasted by the network about the CSI of each user. We first show how to derive the utilities of users that are related to their respective channel quality under the different association policies. We then derive the policy that corresponds to the Stackelberg equilibrium and compare it to the fully cooperative and the non-cooperative model. Technically, our approach not only aims at improving the network equilibrium efficiency but has also two nice features: (i) It allows the network to guide users to a desired equilibrium that optimizes its own utility if it chooses the adequate information to send, (ii) Only the individual user demand and a partial statistical CSI of other users is needed at each transmitter. Our approach contributes to designing networks where intelligence is split between the base station (BS) and mobile stations (MSs) in order to find a desired trade-off between the global network performance reached at the equilibrium and the amount of signaling needed to make it work. Note that the Stackelberg formulation arises naturally in some contexts of practical interest. For example, hierarchy is naturally present in contexts where there are primary (licensed) users and secondary (unlicensed) users who can sense their environment because there are equipped with a radio [7]. It is also natural if the users have access to the medium in an asynchronous manner.

The decision, of which access point to connect to, is typically left to the user. In some cases, each of the access points corresponds to that of another operator. In other cases, the choice of operator is offered to a user only once it connects to an access point. From a practical point of view, the driver for the wireless card typically gives some information concerning the channel state at each of the access points. Figure 1 is an example of the information presented for a user when the opportunity of taking a decision is offered to him. This can be easily thought of as a game situation. This game may involve a preliminary decision to which access point (or service provider) to attempt connection. Once an attempt is made then the user gets information on the pricing policy of the provider. Note that the

user may know the pricing information of one or more providers before making the decision since this pricing usually remains the same for a long period. It may discover the quality of service offered by an operator only after taking the decision of which of the service providers to connect to. In this game, the decisions may depend on the pricing strategy of each service provider as well as on the quality of its service. The latter may be unknown, and become available only after taking, whereas the former may become available. This is a complex stochastic game as each user comes at random points, its decision will be affected by the state of the channel not only at the present (i.e. the one it has available) but also at the future, the latter will be determined by the decisions of future users. Yet, a user is not aware of when future arrivals will occur and what the decisions will be. This game has an unusual information: it is *partial* and *misleading*. Misleading - because, although the channel state indeed can give information on the transmission rate, it is known that the actual throughput of a user is a function of not only his channel state but also of that of the other connected users [8]. The throughput is known to be lower bounded by the harmonic mean of the rates available to each user. The real utility of a user is the throughput he would get and the user may not be aware that it is possible that an access point with a better channel may have a lower throughput because more terminals are connected to it.

The association problem may also include choice between several technologies: say between 3G LTE, WiFi, bluetooth and Ad-Hoc network. Figure 1 is an example of the information available to a user in a game where one has the option of connecting to an Ad-hoc network or to access points with different signal strengths. As before, the information given to the user is misleading since the throughput of the user cannot be directly inferred from the quality of his channel. These questions are tackled in this work where the association problem is modeled as a hierarchical Bayesian game. User strategies are decisions to choose to connect to one system or another according to a local information about the quality of the channel, the demand and a partial statistical information of other users. The operator controls the equilibrium of its wireless users to maximize its own utility by broadcasting appropriate information. We first compute the users' utilities and then derive analytically the utilities related to the channel quality perceived by the users.

RELATED WORK

When we deal with heterogeneous distributed networks, interactions among selfish users sharing a common transmission channel can be modeled as a non-cooperative game using

the game theory framework [3]. Game theory provides a formal framework for studying the interactions of strategic agents. Recently, there has been a surge in research activities that employ game theory to model and analyze a wide range of application scenarios in modern communication networks [9], [10].

The association problem is related in nature to the channel selection problem. We note that when a single technology is used or, when the decision concerns the choice of channels of a given access point rather than the choice of an access point, one can often exploit simpler structure of the decision problem and obtain efficient decentralized solutions. Some examples of work in that direction are [11], [12], [13], [14], [15]. The potential inefficiency of such approaches in the context of 802.11 networks have been known for a long time. The term “*performance anomaly*” has been frequently used for this inefficiency [8]; it describes the fact that when some devices use a lower bit rate than the others, the performance of all devices is considerably degraded. Such a situation is even more problematic when a device attempts to connect to the Internet; it may not be aware that it is possible that an access point with a better channel quality may have a lower throughput because more terminals are connected to it. In fact, this could likely lead the throughput of all devices transmitting at the higher rate degraded below the level of the lower rate. This makes the information given to the user misleading since the throughput of the user cannot be directly inferred from the quality of his channel. To overcome this hurdle, we introduce a Bayesian game theoretic framework with partial CSI to maximize the throughput while taking into account the system overload. This study requires particular attention when all users wish to maximize their individual throughput but each has a different approach (e.g., users may have different tolerance for delay, or may have a certain QoS to guarantee).

The structure of the paper is as follows. The system model related aspects are described in Sec. II. Next in Sec. III, we provide a thorough analysis of the Bayes equilibria for both non-cooperative and Stackelberg frameworks in the case of two-users: Sec. III-A reviews the main results of [6] for the non-cooperative game and Sec. III-B presents the Stackelberg Bayesian game framework adopted for the considered association problem. We first show how the base station can control the equilibrium of its users by means of a Stackelberg formulation and then we derive analytically the utilities of the users and compute equilibria. Sec. III-C presents three different evaluation scenarios along with some key performance indicators

including price of anarchy. In Sec. IV, we generalize the results of previous sections to a situation where there are more than two symmetric users. Sec. V concludes the paper.

II. SYSTEM MODEL

Consider a network composed of Wifi and 3G LTE. Each user entering in the system will decide *individually* to which of the available systems it is best to connect according to its radio condition, its demand and the statistical information about other users. Their policies (or strategies) are then based on this (incomplete) information. The association problem is then generalized to allow the BS to control the users' behavior by broadcasting appropriate information, expected to maximize its utility while individual users maximize their own utility.

We assume that the user state is defined by the pair (h_i, b_i) where h_i is the downlink channel between the WiFi access point (AP) and the terminal and b_i is the demand of user i . The action a_i is defined by the user decision to connect to a certain radio access technology (RAT). The network is fully characterized by the user state. However, when distributing the joint radio resource management (JRRM) decisions, this complete information is not available to the users. The BS or the AP broadcasts to its terminals an aggregated information indicating a measurement of the communication quality of the wireless channel (excellent, fair, poor...). This can be done through the Channel Quality Indicator (CQI) which can be a value (or values) representing a measure of channel quality for a given channel (see Figure 1). Typically, a high value CQI is indicative of a channel with high quality and vice versa. More formally, assume that the knowledge of each user about his own state is limited to the pair (s_i, b_i) , where $s_i = \mathbb{I}_{\{h_i > \Psi_i\}}$, with Ψ – a fixed threshold and \mathbb{I}_C is the indicator function equal to 1 if condition C is satisfied and to 0 otherwise. We will call Ψ_i the “CQI threshold” of user i . Thus, a user only knows whether he wants to transmit and whether the channel is in a good ($s_i = 1$) or in a bad ($s_i = 0$) condition given the CQI threshold. In addition any player has the information about the probability distribution of his own state (s_i, b_i) and that of his opponent (s_j, b_j) . These are given by α_i – the probability to have $\{h_i > \Psi_i\}$, and β_i – the probability that $b_i = 1$.

In the next sections, we provide a thorough analysis of the existence and characterization of the Bayes equilibria for both non-cooperative and Stackelberg scenarios. We first focus on the two-user case in order to gain insights into how to design decision problem in radio environments. Then, we generalize our approach to the multi-user case.

III. THE TWO-USER CASE

The first step before analyzing the Stackelberg Bayesian decision scheme is to define the utilities of users. These are often related to throughput, whose variations are mainly due to network load, radio network conditions and mobility such as handovers. We assume that the 3G LTE network throughput is constant equal to v and that there is no interference between 3G LTE and the WiFi network. Consider the following throughput for each system:

$$Thp_i^W = \log \left(1 + \frac{p h_i a_i b_i}{\sigma^2 + p h_j a_j b_j} \right); \quad j \neq i \quad (1)$$

$$Thp_i^C = v \quad (2)$$

where index W stands for WiFi network and C stands for 3G LTE cellular network. The additive noise variance is σ^2 and p is the transmit power considered constant. We also assume that the distributions of h_i are of exponential type [16]. Given α_i and Ψ_i we can compute that the distribution of h_i is $\text{Exp}(\lambda_i)$ with

$$\lambda_i = -\frac{\log \alpha_i}{\Psi_i}. \quad (3)$$

Given the information that a player has, there are four possible policies of a player i with $b_i = 1$ (we do not consider state $b_i = 0$, when there is no transmission of any type):

$h_i < \Psi_i$	W	W	C	C
$h_i > \Psi_i$	W	C	W	C

Let us not consider the policy (W, C) , which is irrational, as the throughput of a player using WiFi when $\{h_i > \Psi_i\}$ is certainly higher than that when $\{h_i < \Psi_i\}$. We then have a game with partial CSI with two states and a (3×3) matrix in every state.

Let us denote by $\mathbf{P} = [\mathbf{P}_1, \mathbf{P}_2] \in \mathcal{P}$ the (2×2) policy profile matrix defined as the actions taken by the two mobiles in low and high channel states. user i 's utility in state $s = 0, 1$ is then given by

$$u_i(s, \mathbf{P}) = \begin{cases} v; & \text{if user } i \text{ chooses } C \text{ at state } s, \\ C_{\mathbf{P}_j}^i(s); & \text{if user } i \text{ chooses } W \text{ at state } s \end{cases} \quad (4)$$

The functions C_k^i , describing the utility of player i using WiFi when his opponent applies policy k , are defined as follows

$$C_k^i(1) = \mathbb{E}[c_k^i(h_i) | h_i > \Psi_i] = \frac{1}{\alpha_i} \int_{\Psi_i}^{\infty} c_k^i(h_i) \lambda_i e^{-\lambda_i h_i} dh_i, \quad (5)$$

$$C_k^i(0) = \mathbb{E}[c_k^i(h_i)|h_i < \Psi_i] = \frac{1}{1 - \alpha_i} \int_0^{\Psi_i} c_k^i(h_i) \lambda_i e^{-\lambda_i h_i} dh_i, \quad (6)$$

with $k = WW, CW, CC$.

$c_k^i(h_i)$ above is the utility of player i using W when channel gain is h_i against policy k of player j . These utilities are defined as follows:

$$c_{WW}^i(h_i) = \beta_j \int_0^\infty \log\left(1 + \frac{ph_i}{\sigma^2 + ph_j}\right) \lambda_j e^{-\lambda_j h_j} dh_j + \quad (7)$$

$$(1 - \beta_j) \int_0^\infty \log\left(1 + \frac{ph_i}{\sigma^2}\right) \lambda_j e^{-\lambda_j h_j} dh_j$$

Next:

$$c_{CW}^i(h_i) = \beta_j \int_{\psi_j}^\infty \log\left(1 + \frac{ph_i}{\sigma^2 + ph_j}\right) \lambda_j e^{-\lambda_j h_j} dh_j + \quad (8)$$

$$(1 - \beta_j) \int_{\psi_j}^\infty \log\left(1 + \frac{ph_i}{\sigma^2}\right) \lambda_j e^{-\lambda_j h_j} dh_j +$$

$$\int_0^{\psi_j} \log\left(1 + \frac{ph_i}{\sigma^2}\right) \lambda_j e^{-\lambda_j h_j} dh_j$$

Finally:

$$c_{CC}^i(h_i) = \log\left(1 + \frac{ph_i}{\sigma^2}\right) \quad (9)$$

A. Review of the non-cooperative equilibrium

Game theory has accentuated the importance of randomized games or mixed games. However, such a game does not find a significant role in most communication modems and source coding codecs since equilibria where each user randomly picks a decision at each time epoch are unfortunately not interesting in such a case, as they amount to perpetual handover between networks. In what follows, we will make use of the users' utilities obtained above to derive the pure association strategies.

Definition 1 (Bayes-Nash equilibrium). *A strategy profile \mathbf{P}_i^{BNE} , $\forall i = 1, 2$ corresponds to a Bayes-Nash equilibrium (BNE) if, for all users, any unilateral switching to a different strategy cannot improve user's payoff at any state. Mathematically, this can be expressed by the following inequality, given the statistical information about the other user $\forall \mathbf{Q}_i \neq \mathbf{P}_i^{BNE}$*

$$u_i(s_i, (\mathbf{P}_i^{BNE}, \mathbf{P}_{-i}^{BNE})) \geq u_i(s_i, (\mathbf{Q}_i, \mathbf{P}_{-i}^{BNE})) \quad (10)$$

for $s_i = 0, 1$.

Proposition 1. *The game considered in the paper always has a pure-strategy Bayes-Nash equilibrium. Moreover*

- (a) (WW, WW) is an equilibrium iff $C_{WW}^i(0) \geq v$ for $i = 1, 2$.
- (b) (WW, CW) is an equilibrium iff $C_{CW}^1(0) \geq v$ and $C_{WW}^2(0) \leq v \leq C_{WW}^2(1)$.
- (c) (WW, CC) is an equilibrium iff $C_{CC}^1(0) \geq v$ and $C_{WW}^2(1) \leq v$.
- (d) (CW, WW) is an equilibrium iff $C_{WW}^1(0) \leq v \leq C_{WW}^1(1)$ and $C_{CW}^2(0) \geq v$.
- (e) (CW, CW) is an equilibrium iff $C_{CW}^i(0) \leq v \leq C_{CW}^i(1)$ for $i = 1, 2$.
- (f) (CW, CC) is an equilibrium iff $C_{CC}^1(0) \leq v \leq C_{CC}^1(1)$ and $C_{CW}^2(1) \leq v$.
- (g) (CC, WW) is an equilibrium iff $C_{WW}^1(1) \leq v$ and $C_{CC}^2(0) \geq v$.
- (h) (CC, CW) is an equilibrium iff $C_{CW}^1(1) \leq v$ and $C_{CC}^2(0) \leq v \leq C_{CC}^2(1)$.
- (i) (CC, CC) is an equilibrium iff $C_{CC}^i(1) \leq v$ for $i = 1, 2$.

Proof: The statements (a)–(i) are direct consequences of the definition of Bayes-Nash equilibrium and the form of payoff matrices. Next, it is immediate to see that the definitions of $C_k^i(s)$ imply the following inequalities:

$$C_{WW}^i(s) < C_{CW}^i(s) < C_{CC}^i(s)$$

for $i = 1, 2$ and $s = 0, 1$. Now, using these inequalities, it is tedious but straightforward to show that always at least one of the conditions (a)–(i) is satisfied. ■

The next proposition gives us some information on how the Nash-Bayes equilibria depend on the chosen values of the CQI thresholds Ψ_i .

Proposition 2. *If Ψ_1 and Ψ_2 are small enough none of the players uses WW in equilibrium. If they are large enough, none of the players uses policy CC in equilibrium. Moreover, for all the values of the parameters of the model one of the two possibilities is true:*

- (a) For Ψ_1 and Ψ_2 small enough at least one of the players uses policy CC in equilibrium,
- (b) For Ψ_1 and Ψ_2 large enough at least one of the players uses policy WW in equilibrium.

Proof: Define for $i = 1, 2$ and $k = CC, CW, WW$

$$C_k^i(\infty) = \int_0^\infty c_k^i(h_i) \lambda_i e^{-\lambda_i h_i} dh_i \quad (11)$$

Note that when $\Psi_1 \rightarrow 0$ and $\Psi_2 \rightarrow 0$, $C_k^i(0)(\Psi_1, \Psi_2) \rightarrow 0$ and $C_k^i(1)(\Psi_1, \Psi_2) \rightarrow C_k^i(\infty)$ for $i = 1, 2$, $k = CC, CW, WW$. Analogously, when $\Psi_1 \rightarrow \infty$ and $\Psi_2 \rightarrow \infty$, $C_k^i(0)(\Psi_1, \Psi_2) \rightarrow$

$C_k^i(\infty)$ and $C_k^i(1)(\Psi_1, \Psi_2) \rightarrow +\infty$. Thus for Ψ_1 and Ψ_2 small enough, $C_k^i(0)(\Psi_1, \Psi_2) < v$ for all the values of i and k , which by Proposition 1 implies that no player uses policy WW in equilibrium. Analogously for Ψ_1, Ψ_2 big enough, $C_k^i(0)(\Psi_1, \Psi_2) > v$ for all the values of i and k , and thus no player uses CC in equilibrium then.

Now note that by Proposition 1, one of the players uses WW in equilibrium iff

$$C_{CW}^1(0) \geq v \quad \text{or} \quad C_{CW}^2(0) \geq v.$$

Thus if we take Ψ_1, Ψ_2 large enough, we can pass to the limit:

$$C_{CW}^1(\infty) \geq v \quad \text{or} \quad C_{CW}^2(\infty) \geq v. \quad (12)$$

Analogously, one of the players uses CC in equilibrium iff

$$C_{CW}^1(1) \leq v \quad \text{or} \quad C_{CW}^2(1) \leq v.$$

Passing to the limit when Ψ_1, Ψ_2 approach 0,

$$C_{CW}^1(\infty) \leq v \quad \text{or} \quad C_{CW}^2(\infty) \leq v. \quad (13)$$

However (12) and (13) cover all the values of v , ending the proof. ■

Roughly speaking, this proposition means that for higher values of the CQI thresholds Ψ_i s the players are more likely to use WiFi rather than 3G LTE and conversely, for low values of the CQI thresholds Ψ_i s the players are more likely to use 3G LTE rather than WiFi. Interestingly, Proposition 2 also suggests that, rather than increasing the offered throughput v , the operator could control the equilibrium of its wireless users to maximize its own revenue by broadcasting appropriate CQI thresholds. This can lead the network to minimize its overall cost and users to a misleading association problem.

Next, we address a hierarchical approach for choosing the CQI thresholds.

B. The hierarchical equilibrium

In this section, we propose a methodology that transforms the above non-cooperative game into a Stackelberg game. Concretely, the network may guide users to an equilibrium that optimizes its own utility if it chooses the adequate information to send. We first study the policy that maximizes the utility of the network, which is defined as the probability that both

users use 3G LTE network rather than WiFi:

$$\begin{aligned}
U_{BS}(\mathbf{P}, \Psi_1, \Psi_2) = & \\
& (1 - \alpha_1)(1 - \alpha_2)\mathbb{I}_{\{\mathbf{P}_{11}=\mathbf{P}_{21}=C\}} + \alpha_1\alpha_2\mathbb{I}_{\{\mathbf{P}_{12}=\mathbf{P}_{22}=C\}} \\
& + (1 - \alpha_1)\alpha_2\mathbb{I}_{\{\mathbf{P}_{11}=\mathbf{P}_{22}=C\}} + \alpha_1(1 - \alpha_2)\mathbb{I}_{\{\mathbf{P}_{12}=\mathbf{P}_{21}=C\}}
\end{aligned}$$

when α_1, α_2 depend on Ψ_1, Ψ_2 as in (3).

Nevertheless, as it is not realistic to consider that the users will seek the global optimum, we show how to find the policy that corresponds to the Bayes-Stackelberg equilibrium where the BS tries to maximize the probability that both players use 3G LTE network U_{BS} just by choosing the CQI thresholds, knowing that users will try to maximize their individual utility.

Definition 2 (Bayes-Stackelberg equilibrium). By denoting $(\Psi_1^{BSE}, \Psi_2^{BSE})$ the strategy profile of the BS at the Bayes-Stackelberg equilibrium (BSE), this definition translates mathematically as

$$(\Psi_1^{BSE}, \Psi_2^{BSE}) = \arg \max_{\Psi_1, \Psi_2} U_{BS}(\mathbf{P}^{BNE}(\Psi_1, \Psi_2), \Psi_1, \Psi_2), \quad (14)$$

where $\mathbf{P}^{BNE}(\Psi_1, \Psi_2)$ denotes the Bayes-Nash equilibrium in the game of the previous section with CQI thresholds equal to Ψ_1, Ψ_2 .

C. Performance Evaluation

We next exemplify our general analysis by investigating the possibility of considering three scenarios for the choice of Ψ_1 and Ψ_2 :

- 1) *Fully cooperative model* – the base station chooses both Ψ_i s and the policies for the players, aiming to maximize the probability that both players use 3G LTE in the second stage. Formally, the fully cooperative strategy is the one satisfying

$$(\Psi_1^C, \Psi_2^C, \mathbf{P}^C) = \arg \max_{\Psi_1, \Psi_2, \mathbf{P}} U_{BS}(\mathbf{P}, \Psi_1, \Psi_2),$$

- 2) *Stackelberg model* – there are two stages: at the first one the base station chooses both Ψ_i s given the information about the distributions of (h_i, b_i) aiming to maximize the probability that both players use 3G LTE at the second stage, when players play the game from the last section. The proposed approach can be seen as intermediate scheme between the fully cooperative model and the fully non-cooperative model,
- 3) *Fully non-cooperative model* – the game has two stages: at the first one, players choose their Ψ_i s given the information they have about the distributions of (h_i, b_i) aiming to maximize their expected throughput at the second stage; at the second stage they choose

a policy depending on actual (s_i, b_i) as in the model of the last section. Formally, the fully cooperative strategy is the one satisfying

$$\Psi_i^{NC} = \arg \max_{\Psi_i} \mathbb{E}[u_i(s_i, \mathbf{P}^{BNE}(\Psi_i, \Psi_j^{NC}))]; \text{ for } i = 1, 2$$

Below, we analyze the behavior of the base station and the players at the equilibria of each of these models.

Proposition 3.

- 1) In the fully cooperative model, the base station chooses some $\Psi_1 = \Psi_2 = 0$ and CC policies for both users.
- 2) In the Stackelberg model, when¹ $C_{CC}^i(\infty) \leq v$ for $i = 1, 2$ then the base station chooses any $\Psi_1 < \Psi_1^{**}$ and $\Psi_2 < \Psi_2^{**}$ with Ψ_i^{**} satisfying² $C_{CC}^1(1)(\Psi_1^{**}) = v$ and $C_{CC}^2(1)(\Psi_2^{**}) = v$ and then users both play CC.

When $C_{CC}^i(\infty) > v$ for $i = 1$ or $i = 2$, then the base station chooses Ψ_i^{***} , for $i = 1, 2$ maximizing³ either

$$(1 - e^{-\lambda_1 \Psi_1})(1 - e^{-\lambda_2 \Psi_2})$$

subject to $C_{CW}^i(0)(\Psi_1, \Psi_2) \leq v$ $i = 1, 2$ or maximizing

$$1 - e^{-\lambda_i \Psi_i}$$

subject to $C_{CC}^i(0)(\Psi_1, \Psi_2) \leq v$ and $C_{CW}^j(1)(\Psi_1, \Psi_2) \leq v$. In the first case, both players choose CW in the second stage. In the second case, user i chooses CW and user j chooses CC.

- 3) In the fully non-cooperative model, the players in equilibrium choose $\Psi_1 = \Psi_1^*$ and $\Psi_2 = \Psi_2^*$ satisfying

$$c_{CW}^1(\Psi_1^*) = v = c_{CW}^2(\Psi_2^*) \quad (15)$$

and then both use a CW policy.

For the clarity of the exposition, proofs are given in the Appendix. What we see in this proposition is that when the BS can decide on the behavior of the users, it chooses not to disclose any additional information to them by giving $\Psi_1 = \Psi_2 = 0$ and forces them to

¹Recall definition (11).

²Here and in the sequel $C_k^i(s)(\Psi_1, \Psi_2)$ denotes the respective $C_k^i(s)$ when the values of Ψ_i s have the given value.

³Of course the one of the two with the higher objective function is chosen – its value is the BS utility at equilibrium.

use 3G LTE. In other cases (when users can decide on their behavior, but are given only partial information), the users' interest is to choose the CQI thresholds somewhere in the middle of the channel gain range. This can be seen as a desired trade-off between the global network performance at the equilibrium and the individual efficiency of all the users. On the other hand, the BS has an incentive to choose CQI thresholds either very low (first case in the Stackelberg scenario) or very high (the second case). Both these choices give little information for the user about actual channel condition, which is precisely what he wants to avoid. It is interesting and somewhat surprising that the optimal policy of the BS in the Stackelberg game can be both giving high or low values of CQI thresholds. This can however be explained when we understand the meaning of these two situations – very low value of the threshold means that no information about the channel state is given. In this case, when both users connect to 3G LTE, this corresponds to the choice of the BS. Now, if in the "no information" case players choose WiFi, then the base station tries to divide the range of h_i into a small (in terms of probability) part when the players use WiFi and, a large one when they use 3G LTE. This is done by giving the highest possible CQI threshold below which the players would have an incentive to use rather 3G LTE than WiFi. This explains why the BS has an incentive to choose CQI thresholds very high in this case.

The final two results of this section are given without proofs, which are straightforward.

Corollary 1. *Note that the maximum network utility, obtained in scenario 1) is equal to 1. Obviously the utilities obtained in the other two scenarios always satisfy*

$$\begin{aligned} 1 &\geq U_{BS}(\mathbf{P}^{BSE}, \Psi_i^{BSE}, \Psi_j^{BSE}) > \\ U_{BS}(\mathbf{P}^{NC}, \Psi_i^{NC}, \Psi_j^{NC}) &> 0 \end{aligned} \tag{16}$$

The important fact the corollary implies is that the users, if choosing thresholds optimal from their point of view, never choose the same way the base station would, but their interests are not contradicting. Note that by Proposition 2 there exists a choice of thresholds Ψ_1 and Ψ_2 which gives a zero utility for the base station. The users never have incentive to make such a choice.

In the second corollary, we give the method to compute the price of anarchy (PoA) for our model. The PoA measures how good the system performance is when users play selfishly and reach the NE instead of playing to achieve the social optimum [17][18]. Note that as

the maximum network utility which can be obtained is 1, the price of anarchy when players use strategy profile \mathbf{P} is

$$PoA = \frac{1}{U_{BS}(\mathbf{P}, \Psi_1, \Psi_2)}.$$

Thus, Proposition 3 implies that

Corollary 2. *The price of anarchy in the Stackelberg model equals 1 whenever $C_{CC}^i(\infty) \leq v$ for $i = 1, 2$. When for some i , $C_{CC}^i(\infty) > v$, then the price of anarchy is equal to the smaller of the two values:*

$$\min_{C_{CW}^k(0)(\Psi_1, \Psi_2) \leq v, k=1,2} \frac{1}{(1 - e^{-\lambda_1 \Psi_1})(1 - e^{-\lambda_2 \Psi_2})},$$

$$\min_{C_{CC}^i(0)(\Psi_1, \Psi_2) \leq v, C_{CW}^j(1)(\Psi_1, \Psi_2) \leq v} \frac{1}{1 - e^{-\lambda_i \Psi_i}}.$$

In fully non-cooperative model,

$$PoA = \frac{1}{(1 - e^{-\lambda_1 \Psi_1^*})(1 - e^{-\lambda_2 \Psi_2^*})},$$

where Ψ_1^*, Ψ_2^* satisfy (15).

The above corollary is just a rewriting of the Proposition 3 using different language. It is however important to see that the algorithms of finding equilibria for all the hierarchical models considered give us not only the equilibrium strategies, but also the tools to evaluate the performance of the network in each of these situations.

IV. THE MULTI-USER CASE

Now let us consider the case where instead of two we have n users choosing to connect either to WiFi or to 3G LTE network. Again we assume that the information about the channel quality that user i possesses is limited to that about the distributions of states (s_j, b_i) of each of the players (including i), that is about α_j (or λ_j) and β_j and to exact information about his own current state (s_i, b_i) (but not about exact value of h_i). Our additional assumption about the model considered in this section is that the model is symmetric, that is all the values β_i, λ_i and Ψ_i defining it, are the same for each of the players (and equal to β, λ and Ψ respectively). This significantly simplifies the notation without any serious limitation of generality (we believe that some counterparts of all our results will be true also for asymmetric model).

To define the utilities of the players first let us redefine throughput for each system:

$$Thp_i^W = \log \left(1 + \frac{p h_i a_i b_i}{\sigma^2 + p \sum_{j \neq i} h_j a_j b_j} \right) \quad (17)$$

$$Thp^C = v \quad (18)$$

Again we assume that each of the players uses one of the three policies WW, CW, CC , where first letter stands for a player's action when his channel is bad, and the second one when his channel is good. As it is troublesome to write down the policies for each of n players, we will make use of the fact that the game is symmetric, writing instead of the policy profile a policy statistics $\mathbf{K} = (k_{WW}, k_{CW})$ with k_{WW} denoting the number of players applying policy WW and k_{CW} – of players applying CW . Of course the number of those using policy CC is $n - k_{WW} - k_{CW}$, so we will omit it. Given \mathbf{K} , we can define user i 's utility in state $s = 0, 1$ as⁴

$$u_i(s, \mathbf{K}) = \begin{cases} v; & \text{if user } i \text{ chooses } C \text{ at state } s, \\ C_{\mathbf{K}_{-i}}^i(s); & \text{if user } i \text{ chooses } W \text{ at state } s \end{cases} \quad (19)$$

where the functions $C_{\mathbf{K}_{-i}}^i$, describing the utility of player i using WiFi when his opponents use policies described by \mathbf{K} , are similarly as for the two-user case:

$$C_{\mathbf{K}_{-i}}^i(1) = \mathbb{E}[c_{\mathbf{K}_{-i}}^i(h_i) | h_i > \Psi] \quad (20)$$

$$= \frac{1}{\alpha_i} \int_{\Psi_i}^{\infty} c_{\mathbf{K}_{-i}}^i(h_i) \lambda_i e^{-\lambda_i h_i} dh_i,$$

$$C_{\mathbf{K}_{-i}}^i(0) = \mathbb{E}[c_{\mathbf{K}_{-i}}^i(h_i) | h_i < \Psi] \quad (21)$$

$$= \frac{1}{1 - \alpha_i} \int_0^{\Psi_i} c_{\mathbf{K}_{-i}}^i(h_i) \lambda_i e^{-\lambda_i h_i} dh_i.$$

Next, the functions $c_{\mathbf{K}_{-i}}^i$, defining utility of player i using W when channel gain is h_i against

⁴Notation \mathbf{K}_{-i} used below denotes policy statistics defined as in the two-user case but without policy of user i .

policies \mathbf{K} of his opponents, can be written as⁵:

$$c_{[k_1, k_2]}^i(h) = \sum_{r=0}^{k_1} \sum_{q=0}^{k_2} \sum_{v=0}^q \beta^{r+q} (1-\beta)^{k_1+k_2-r-q} \binom{k_1}{r} \binom{k_2}{q} \quad (22)$$

$$\binom{q}{v} \int_{[\psi, \infty)^v \times \mathbb{R}^{n-q-1} \times [0, \Psi)^{q-v}} e^{-\lambda \sum_{j=1}^{n-1} h_j}$$

$$\log(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r+v} h_j}) \lambda^{n-1} dh_1 \dots dh_{n-1}$$

Below we give a generalization of Proposition 1 for the n -user case.

Proposition 4. *The symmetric n -user game considered in the paper always has a pure-strategy Bayes-Nash equilibrium of one of five types:*

- (a) *When $C_{[0,0]}^i(1) \leq v$ then the profile where all the players use policy CC is an equilibrium.*
- (b) *When $C_{[0,k-1]}^i(1) \geq v \geq C_{[0,k]}^i(1)$ then any profile where k players use policy CW and all the others play CC is an equilibrium.*
- (c) *When $C_{[0,n-1]}^i(1) \geq v$ and $C_{[0,n-1]}^i(0) \leq v$ then the profile where all the players use policy CW is an equilibrium.*
- (d) *When $C_{[k,n-k-1]}^i(1) \geq v$ and $C_{[k-1,n-k]}^i(0) \geq v \geq C_{[k,n-k]}^i(0)$ then any profile where k players apply policy WW and remaining $n-k$ players use policy CW is an equilibrium.*
- (e) *When $C_{[0,n-1]}^i(0) \geq v$ then the profile where all the players use policy WW is an equilibrium.*

It may also have another pure-strategy Bayes-Nash equilibrium with k players using WW and $l < n-k$ using CW when $C_{[k,l-1]}^i(1) \geq v \geq C_{[k,l]}^i(1)$ and $C_{[k-1,l]}^i(0) \geq v \geq C_{[k,l]}^i(0)$.

We give two corollaries of this proposition. The first one considering two-user games discussed before is immediate.

Corollary 3. *Whenever the two-user symmetric game has an equilibrium of the form (CC, WW) or (WW, CC), it also has another pure equilibrium, where in at least one state both players use the same action.*

The second corollary gives a kind of consistency property for equilibria in games for different values of n .

⁵Of course this formula is a generalization of the formulas for c_k^i given in section III and it applies for any $n \geq 2$, in particular $c_{CC}^i \equiv c_{[0,0]}^i$, $c_{CW}^i \equiv c_{[0,1]}^i$ and $c_{WW}^i \equiv c_{[1,0]}^i$ when $n = 2$ and players are symmetric.

Corollary 4. *Suppose that*

- (a) *a profile where k players apply policy CW and all the other players use policy CC is an equilibrium in n -user symmetric game. Then it is also an equilibrium in any m -user game defined with the same parameters β, λ and Ψ and $m \geq k$.*
- (b) *a profile where k players use policy WW, l players use policy CW and $k + l < n$ is an equilibrium in n -user symmetric game. Then it is also an equilibrium in any m -user game defined with the same parameters β, λ and Ψ and $m \geq k + l$.*

Moreover for any parameters β, λ and Ψ there exists an n such that for any $m > n$ any profile with n players applying policy CW and $m - n$ using policy CC is an equilibrium in m -user game.

Proof: Note that $C_{[k_1, k_2]}^i(s)$ does not depend on the number of players in the game n , only on the number of those who use one of the policies CW or WW. Just this implies parts (a) and (b). The final part is due to the fact that $C_{[0, n-1]}^i(1) \rightarrow 0$ as $n \rightarrow \infty$. ■

The next proposition generalizes the results for hierarchial model included in Proposition 3 for n -user symmetric games. We only consider scenarios 1) and 2) discussed there, as it is difficult to apply scenario 3) to the symmetric model.

Proposition 5.

- 1) *In the fully cooperative model, the base station chooses $\Psi = 0$ and CC policies for all the users.*
- 2) *In the Stackelberg model the base station computes for every $k \leq n$ such that⁶*

$$C_{[k-1]}^i(\infty) := \int_0^\infty c_{[k-1, 0]}^i(h) \lambda e^{-\lambda h} dh \geq v$$

$\Psi(k)$ such that

$$C_{[0, k-1]}^i(0)(\Psi(k)) = v.$$

Then for each such k it computes

$$P(k) = (1 - e^{-\lambda \Psi(k)})^k$$

and chooses k^ with the biggest value of $P(k)$ (which equals the BS utility at equilibrium). The choice of $\Psi(k^*)$ at the first stage and any profile of policies where k^* players use policy CW and all the remaining ones play CC will then be an equilibrium.*

⁶There will always be only a finite number of k satisfying this inequality, as $\int_0^\infty c_{[k-1, 0]}^i(h) \lambda e^{-\lambda h} dh \rightarrow 0$ for $k \rightarrow \infty$.

If even $C_{[0]}^i(\infty) < v$, then at the equilibrium the base station chooses any Ψ such that $C_{[k-1,0]}^i(1)(\Psi) < v$ and all the players use policy CC.

We give one corollary to this proposition.

Corollary 5. *The price of anarchy in the n -user Stackelberg model can be computed as*

$$PoA = \frac{1}{U_{BS}(\mathbf{K}, \psi)}$$

(where the base station utility U_{BS} is generalized from the two-user case as the probability that all the users choose rather 3G LTE than WiFi), and is either equal to 1, when $C_{[0]}^i(\infty) < v$, or satisfies

$$PoA = \min_{k \leq n, k \leq n^*} \frac{1}{P(k)}$$

with $n^* = \max\{k : C_{[k-1]}^i(\infty) \geq v\}$, when $C_{[0]}^i(\infty) \geq v$. For any given values of β and λ , the price of anarchy is a non-increasing function of n and constant for $n \geq n^*$.

The first part of this corollary is again just a rewriting of the results from Proposition 5 with the stress made on network utilities rather than strategies of the players. It shows that exactly the same procedure, used to find the equilibrium policies, can be applied to evaluate the performance of the network.

The second part of the corollary (the properties of the PoA) is a consequence of the fact that adding each new player to the game gives the BS more patterns of behavior of the users which can be stimulated by a proper choice of Ψ . This results in the decrease in the PoA up to the threshold number of players n^* from which no new player in the game is interested in using WiFi network, because for such a large number of players it would be too slow, regardless of how good the channel would be. This result may seem surprising at first glance, as usually a bigger number of players means more anarchy. However, if we look at the objective function of the BS, which is the probability of all the players using 3G LTE network, we clearly see that a bigger number of players is disadvantageous for the WiFi network which may get congested and favorable for 3G LTE which cannot.

If someone is interested not in finding the equilibria for all the numbers of players, but only in the limit behavior and the limit value of PoA without checking the values of $C_{[k-1]}^i(\infty)$ for every k , he may, instead of computing n^* given above compute an upper bound given below.

Corollary 6. *The value n^* appearing above can be bounded from above by $n^{**} = \max\{\frac{2}{\beta} + 1, k^*\}$ with k^* satisfying*

$$\frac{\lambda}{2} e^{-\frac{(k^*-1)\beta^2}{2}} \int_0^\infty \log(1 + \frac{ph}{\sigma^2}) e^{-\lambda h} dh + \log(\frac{(k^*-1)\beta}{(k^*-1)\beta - 2}) = v.$$

This last bound cannot be given in a closed form, but it can be computed much faster than n^* .

V. CONCLUSION

We have proposed a hierarchical association method that combines benefits from both decentralized and centralized design in which the network operator optimizes its global utility while users maximize their individual utilities. The users' decision making is based on partial information that is signaled to the mobiles by the base station. A central design aspect is then for the base stations to decide how to aggregate information which then determines what to signal to the users. In this setting, we have shown that, in order to maximize its revenue, the network operator rather than increasing its offered throughput (which is costly) has an incentive to choose channel quality indicator thresholds either very low or very high. This may make the information given to the user when attempting to connect *misleading* since the throughput of a user cannot be directly inferred from the quality of his channel but also depends on the channel quality indicator thresholds the base station broadcasts. In particular, there may be different equilibria (so different outcomes) depending on what information (channel quality indicator thresholds) the base station broadcasts to users.

The proposed approach provides a reasonable trade-off between centralized vs decentralized optimization in terms of the signaling overhead and the resulting network throughput performance.

APPENDIX

A. Proof of Proposition 3

Proof:

- 1) is obvious and needs no explanation.
- 2) Since when $\Psi_1 \rightarrow 0$ and $\Psi_2 \rightarrow 0$, $C_{CC}^i(1)(\Psi_1, \Psi_2) \rightarrow C_{CC}^i(\infty)$ for $i = 1, 2$, when $C_{CC}^i(\infty) \leq v$, then for Ψ_i small enough (in the worst case, equal to 0) also $C_{CC}^i(1)(\Psi_1, \Psi_2) \leq v$ for $i = 1, 2$. But this means that (CC, CC) is an equilibrium in the game at the second stage. Thus whenever $\Psi_1 < \Psi_1^{**}$ and $\Psi_2 < \Psi_2^{**}$ with Ψ_i^{**} satisfying $C_{CC}^i(1)(\Psi_1^{**}, \Psi_2^{**}) = v$, the

outcome of the Stackelberg game is that both players use 3G LTE with probability 1, which is the biggest value possible of the base station's utility. Now suppose that $C_{CC}^i(\infty) > v$. Then even for the value of Ψ_i equal to 0, playing (CC, CC) is not an equilibrium in the game of the second stage. Thus, to maximize the probability of both players using 3G LTE, the base station has to choose the Ψ_i and Ψ_j in such a way that the equilibrium in the game of the second stage was either (CW, CW) or (CW, CC) and that the probability that the state of players using policy CW is 0 ($h_i < \Psi_i$) is the highest possible. This is done by solving the optimization problems defined in the proposition (the first problem for the case (CW, CW) , the second one for (CW, CC)).

3) First note that whenever Ψ_1^* and Ψ_2^* are chosen as in (15), (CW, CW) is an equilibrium. This is because $C_{CW}^i(0)$ is a conditional expectation of $c_{CW}^i(h_i)$ over the set $H_- := \{c_{CW}^i(h_i) \leq v\}$, so it is definitely smaller than v . Similarly, $C_{CW}^i(1)$ is a conditional expectation of $c_{CW}^i(h_i)$ over the set $H_+ := \{c_{CW}^i(h_i) \geq v\}$, so it is bigger than v . Thus the condition for (CW, CW) to be an equilibrium is definitely satisfied.

Now note that whenever player i chooses $\Psi_i < \Psi_i^*$ at the first stage, but continues to use policy CW in the second, he loses

$$\int_{\Psi_i}^{\Psi_i^*} (v - c_{CW}^i(h_i)) \lambda_i e^{-\lambda_i h_i} > 0.$$

Similarly, when he chooses $\Psi_i > \Psi_i^*$ he loses

$$\int_{\Psi_i^*}^{\Psi_i} (c_{CW}^i(h_i) - v) \lambda_i e^{-\lambda_i h_i} > 0.$$

On the other hand, when he changes both the Ψ_i and the policy at the second stage, his utility is either v (when he plays CC) or $\mathbb{E}[c_{CW}^i(h_i)]$ (when he uses policy WW), which are clearly both less than his current utility

$$P(h_i \in H_-)v + P(h_i \in H_+)\mathbb{E}[c_{CW}^i(h_i)|h_i \in H_+],$$

so (Ψ_1^*, Ψ_2^*) is an equilibrium choice.

The last thing we need to show is that there are Ψ_1^* and Ψ_2^* satisfying (15). Note however that if we construct functions⁷ $\widehat{\Psi}_i(\Psi_j) := \{\Psi_i : c_{CW}^i(\Psi_i)(\Psi_j) = v\}$. It is immediate to see that since all the functions c_k^i are nondecreasing, $\widehat{\Psi}_i$ are non-increasing functions from $[0, \infty)$

⁷Here we use a convention that in the second bracket we give the value of player j 's threshold, which appears in the definitions of c_k^i , but was omitted so far.

to itself. It is then obvious that the graphs of these two functions: $\{(\Psi_1, \Psi_2) : \Psi_1 = \widehat{\Psi}_1(\Psi_2)\}$ and $\{(\Psi_1, \Psi_2) : \Psi_2 = \widehat{\Psi}_2(\Psi_1)\}$ intersect, and thus (15) has a solution. ■

B. Proof of Proposition 4

Before we prove Proposition 4, we need an auxiliary lemma.

Lemma 1. *The functions $C_{[k,l]}^i(s)(\Psi)$ are decreasing in k , l and increasing in Ψ for any $s = 0, 1$.*

Proof: First note that

$$F(r, q) := \sum_{v=0}^q \binom{q}{v} \int_{[\psi, \infty)^v \times \mathbb{R}^{n-q-1} \times [0, \Psi)^{q-v}} \lambda^{n-1} \log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r+v} h_j}\right) e^{-\lambda \sum_{j=1}^{n-1} h_j} dh_1 \dots dh_{n-1}$$

is decreasing in r .

On the other hand $F(r+1, q) \geq F(r, q+1)$, as

$$\begin{aligned} F(r, q+1) &= \sum_{v=0}^{q+1} \binom{q+1}{v} \int_{[\psi, \infty)^v \times \mathbb{R}^{n-q-2} \times [0, \Psi)^{q-v+1}} \lambda^{n-1} \log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r+v} h_j}\right) e^{-\lambda \sum_{j=1}^{n-1} h_j} dh_1 \dots dh_{n-1} \\ &= \sum_{v=0}^q \binom{q}{v} \int_{[\psi, \infty)^v \times \mathbb{R}^{n-q-2} \times [0, \Psi)^{q-v}} \lambda^{n-2} \cdot \left[\int_0^\Psi \log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r+v} h_j}\right) \lambda e^{-\lambda h_{r+v+1}} dh_{r+v+1} \right. \\ &\quad \left. + \int_\Psi^\infty \log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r+v+1} h_j}\right) \lambda e^{-\lambda h_{r+v+1}} dh_{r+v+1} \right] e^{-\lambda \sum_{j=1}^{n-2} h_j} dh_1 \dots dh_{r+v} dh_{r+v+2} \dots dh_{n-1} \\ &\geq \sum_{v=0}^q \binom{q}{v} \int_{[\psi, \infty)^v \times \mathbb{R}^{n-q-2} \times [0, \Psi)^{q-v}} \lambda^{n-2} \int_0^\infty \log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r+v+1} h_j}\right) \lambda e^{-\lambda h_{r+v+1}} dh_{r+v+1} \\ &\quad e^{-\lambda \sum_{j=1}^{n-2} h_j} dh_1 \dots dh_{r+v} dh_{r+v+2} \dots dh_{n-1} = \sum_{v=0}^q \binom{q}{v} \int_{[\psi, \infty)^v \times \mathbb{R}^{n-q-1} \times [0, \Psi)^{q-v}} \lambda^{n-1} \\ &\quad \log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r+v+1} h_j}\right) e^{-\lambda \sum_{j=1}^{n-1} h_j} dh_1 \dots dh_{n-1} \\ &= F(r+1, q) \end{aligned}$$

But this implies that F is also decreasing in q .

Next note that $c_{[k,l]}^i(h)$ is the expected value of $\sum_{q=0}^l \beta^q (1-\beta)^{l-q} \binom{l}{q} F(r, q)$ when r is a random value with the binomial distribution $\text{Bin}(k, \beta)$. As distribution $\text{Bin}(k+1, \beta)$ strictly stochastically dominates $\text{Bin}(k, \beta)$, the expected value with respect to $\text{Bin}(k+1, \beta)$ of any decreasing function is smaller than that with respect to $\text{Bin}(k, \beta)$ and thus

$$c_{[k+1,l]}^i(h) < c_{[k,l]}^i(h)$$

for any $h \geq 0$. But, as $C_{[k,l]}^i(0)$ and $C_{[k,l]}^i(1)$ are conditional expectations of $c_{[k,l]}^i(h)$ over some fixed sets, this immediately implies that they are both decreasing in k . The fact that they are decreasing in l is proved analogously – the only difference is that the monotonicity of F in q (instead of the monotonicity in r) is used.

To prove the last part of the lemma take $\Psi_1 < \Psi_2$ and define for any q and $\alpha \in \{0, 1, 2\}^q$

$$\begin{aligned} S_\alpha(\Psi_1, \Psi_2) &= \{(h_1, \dots, h_{n-1}) \in \mathbb{R}^{n-1} : \\ 0 &\leq h_j < \Psi_1 \text{ if } \alpha_j = 0, \Psi_1 \leq h_j < \Psi_2 \text{ if } \alpha_j = 1, \\ \Psi_2 &\leq h_j \text{ if } \alpha_j = 2\}. \end{aligned}$$

Note that $\mathbb{R}^{n-1} = \bigcup_\alpha S_\alpha(\Psi_1, \Psi_2)$. Next note that $F(r, q)(\Psi_1)$ is the integral over \mathbb{R}^{n-1} of the function f_1 defined on each $S_\alpha(\Psi_1, \Psi_2)$ separately, as

$$\log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j \geq 1} h_j}\right) \lambda^{n-1} e^{-\lambda \sum_{j=1}^{n-1} h_j}.$$

On the other hand $F(r, q)(\Psi_2)$ is the integral over \mathbb{R}^{n-1} of the function f_2 defined on each $S_\alpha(\Psi_1, \Psi_2)$ separately, as

$$\log\left(1 + \frac{ph}{\sigma^2 + p \sum_{j \geq 2} h_j}\right) \lambda^{n-1} e^{-\lambda \sum_{j=1}^{n-1} h_j}.$$

Clearly $f_1 < f_2$, and so $F(r, q)(\Psi_1) < F(r, q)(\Psi_2)$ for any r and q . This immediately implies that also $c_{[k,l]}^i(h)(\Psi_1) < c_{[k,l]}^i(h)(\Psi_2)$. However, note that since $c_{[k,l]}^i(h)$ are also increasing in h : Similarly

$$\begin{aligned} C_{[k,l]}^i(0)(\Psi_1) &= \mathbb{E}[c_{[k,l]}^i(h)(\Psi_1) | h < \Psi_1] \\ &\leq \mathbb{E}[c_{[k,l]}^i(h)(\Psi_1) | h < \Psi_2] \\ &< \mathbb{E}[c_{[k,l]}^i(h)(\Psi_2) | h < \Psi_2] \\ &= C_{[k,l]}^i(0)(\Psi_2), \end{aligned}$$

which ends the proof of lemma. ■

Now we are able to prove Proposition 4.

Proof: First note that it is clear from the definition of $c_{[k,l]}^i(h)$ that it is increasing in h and thus

$$C_{[k,l]}^i(0) < C_{[k,l]}^i(1) \quad (23)$$

for any values of k and l . Next it is enough to check the definition of Bayes-Nash equilibrium (inferring (23) if needed) that the sets of inequalities appearing in the proposition define respective equilibria. What is left to show is that cases (a–e) cover all the possible situations. Suppose that none of the cases (a) and (b) holds. Then, since $C_{[0,0]}^i(1) > v$, by Lemma 1 also $C_{[0,n-1]}^i(1) > v$. However, (c), (d) and (e) cover all the possible cases then. ■

C. Proof of Proposition 5

Proof:

Part 1) is obvious. 2) Since when $\Psi \rightarrow 0$, $C_{[0,0]}^i(1)(\Psi) \rightarrow C_{[0]}^i(\infty)$, then when $C_{[0]}^i(\infty) \leq v$, for Ψ small enough (in the worst case, equal to 0) also $C_{[0,0]}^i(1)(\Psi) \leq v$, which this means that all the players apply policy CC in equilibrium at the second stage of the game. Thus whenever Ψ is small enough, the outcome of the Stackelberg game is that all the players use 3G LTE with probability 1, which is the biggest value possible of the base station's utility.

Now suppose that $C_{[0]}^i(\infty) > v$. Then even for the value of Ψ equal to 0, not every player uses policy CC at the equilibrium of the game of the second stage. Thus, to maximize the probability of both players using 3G LTE, the base station has to choose the Ψ in such a way that at the equilibrium of the game of the second stage all of the players applied either policy CC or CW and that the probability that the state of players using policy CW is 0 ($h_i < \Psi$) is the highest possible. This is done by solving the optimization problems of finding Ψ maximizing $(1 - e^{-\lambda\Psi})^k$ with respect to $C_{[0,k-1]}^i(0)(\Psi) \leq v$. However, as by Lemma 1 $C_{[0,k-1]}^i(0)(\Psi)$ is an increasing function of Ψ for any fixed k , this maximum is achieved for Ψ satisfying $C_{[0,k-1]}^i(0)(\Psi) = v$, which ends the proof. ■

D. Proof of Corollary 6

Proof:

Let us assume that

$$k > \frac{2}{\beta} + 1. \quad (24)$$

First note that $C_{[k-1]}^i(\infty)$ is

$$\sum_{r=0}^{k-1} \beta^r (1-\beta)^{k-1-r} \binom{k-1}{r} \mathbb{E}[\log(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^r h_j})] \quad (25)$$

where h and h_1, \dots, h_{k-1} are independent exponentially distributed random values with common parameter λ .

Next let $r^* = \frac{(k-1)\beta}{2}$. Now (25) can be rewritten as

$$\begin{aligned} & \sum_{r < r^*} \beta^r (1-\beta)^{k-1-r} \binom{k-1}{r} \mathbb{E}[\log(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^r h_j})] \\ & + \sum_{r \geq r^*} \beta^r (1-\beta)^{k-1-r} \binom{k-1}{r} \mathbb{E}[\log(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^r h_j})]. \end{aligned}$$

Since the function $\mathbb{E}[\log(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^r h_j})]$ is clearly positive decreasing, the first element of this sum can be bounded from above by

$$\text{Prob}[r < r^*] E[\log(1 + \frac{ph}{\sigma^2})],$$

where $\text{Prob}[r < r^*]$ is the probability that a random value with binomial distribution $\text{Bin}(k-1, \beta)$ is smaller than r^* . This probability, using Hoeffding's inequality [19] can be bounded above by $\frac{1}{2} e^{-\frac{(k-1)\beta^2}{2}}$ and thus the whole term by $\frac{1}{2} e^{-\frac{(k-1)\beta^2}{2}} E[\log(1 + \frac{ph}{\sigma^2})]$.

Analogously, the second element of the sum can be bounded from above by

$$\text{Prob}[r \geq r^*] \mathbb{E}[\log(1 + \frac{ph}{\sigma^2 + p \sum_{j=1}^{r^*} h_j})]$$

and further by

$$\mathbb{E}[\log(1 + \frac{h}{\sum_{j=1}^{r^*} h_j})]. \quad (26)$$

Now note that $1 + \frac{h}{\sum_{j=1}^{r^*} h_j}$ is a random value with Pareto distribution [20, Chap. 20, Sec. 12] with parameters 1 and r^* , whose average is (for $r^* > 1$, which is guaranteed by our assumption (24)) $\frac{r^*}{r^*-1}$. Since logarithm is a concave function, we can use Jensen's inequality to bound (26) from above by

$$\log(\frac{r^*}{r^*-1}) = \log(\frac{(k-1)\beta}{(k-1)\beta-2}).$$

This implies that

$$C_{[k-1]}^i(\infty) < \frac{\lambda}{2} e^{-\frac{(k-1)\beta^2}{2}} \int_0^\infty \log(1 + \frac{ph}{\sigma^2}) e^{-\lambda h} dh + \log(\frac{(k-1)\beta}{(k-1)\beta-2})$$

and consequently that for any k such that the RHS of the above inequality equals v the LHS will be smaller than v and thus $k > n^*$. ■

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